

1. Consider this problem: *The average age of the 48 employees at Cooper Inc is 51 years. Six people retire; their ages are 69, 68, 62, 62, 64, and 65. What is the average age of the remaining employees?*

(a) *[3 pts]* List three problem-solving strategies that you could try in solving it.

(b) *[9 pts]* Solve the problem, showing clear work. You need not explain.

2. *[4 pts]* Eldon has saved \$48.60 toward a new game, which is  $\frac{2}{3}$  of what it costs. What is the price of the game? Show clear work, but you need not explain.

3. (a) *[3 pts]* Find the first five terms of a geometric sequence whose third term is 180 and whose common ratio is  $\frac{1}{3}$ .

(b) *[3 pts]* Find the 298th term of this sequence: 140, 143, 146,  $\dots$

4. *[8 pts]* Demonstrate Gauss' Trick in computing this sum:

$$526 + 535 + 544 + \dots + 2209$$

5. [6 pts] Find a formula for the  $n$ th term of this sequence: 3, 6, 11, 18, ... Show clear work.

6. For this entire problem, let  $U = \{1, 2, 3, \dots, 20\}$ , and let  $A = \{x \mid x \text{ is a multiple of } 6\}$ ,  $B = \{1, 2, 7, 18, 19\}$ , and  $C = \{1, 19, 20\}$ .

(a) [2 pts] Rewrite  $A$  using correct listing notation.

(b) [6 pts] How many subsets does  $B$  have? Use correct notation to list two of them having different cardinalities.

(c) [4 pts] Use correct listing notation to find  $\overline{B} \cap C$ .

(d) [2 pts] Use correct notation to write a single element of  $C \times B$ .

(e) [6 pts - 2 each] Classify each statement below as true or false:

i.  $\emptyset \in A$

ii.  $2 \in B \cup C$

iii.  $\{6\} \subseteq A$

7. [8 pts - 2 each] Fill in each blank with the correctly spelled term.

(a) In the number sentence,  $8 - 5 = 3$ , the number 5 is called the

\_\_\_\_\_

while the answer, 3, is called the

\_\_\_\_\_.

(b) In the number sentence  $2 + 9 = 11$ , the numbers 2 and 9 are called the

\_\_\_\_\_.

(c) In the number sentence  $42 \div 6 = 7$ , the number 42 is called the

\_\_\_\_\_.

8. [4 pts] Circle the computation that *cannot* be performed, then clearly explain why, referring to definitions or models that we have studied.

$$0 \div 8$$

$$8 \div 0$$

9. [6 pts - 2 each] Write the complete name (for instance, “\_\_\_\_\_ Property of Multiplication”) of the property best illustrated in each number sentence below. Spell correctly.

(a)  $3 \cdot (2 + 5) + (4 \cdot 6) \cdot 7 = 3 \cdot (2 + 5) + 4 \cdot (6 \cdot 7)$

(b)  $3 \cdot (2 + 5) + (4 \cdot 6) \cdot 7 = 0 + 3 \cdot (2 + 5) + (4 \cdot 6) \cdot 7$

(c)  $3 \cdot (2 + 5) + (4 \cdot 6) \cdot 7 = 3 \cdot (2 + 5) + 7 \cdot (4 \cdot 6)$

10. (a) [3 pts] List the three numbers that *precede*  $501_{seven}$  in base seven.

(b) [3 pts] List the three numbers that *follow*  $8T9_{twelve}$  in base twelve.

11. [4 pts] Convert:  $5014_{\text{twenty-three}} = \underline{\hspace{4cm}}_{\text{ten}}$ .

12. [5 pts] Demonstrate “balancing subtraction” to compute  $7654_{\text{sixteen}} - 213C_{\text{sixteen}}$  entirely in base sixteen.

13. [8 pts] Compute entirely in base five:  $3004_{\text{five}} \div 24_{\text{five}}$ . Show clear work.

14. [8 pts] Multiply entirely in base eight using your choice of algorithm:  $562_{\text{eight}} \times 435_{\text{eight}}$ .
15. (a) [3 pts] Write 629 as a Roman numeral.
- (b) [4 pts] Create the smallest possible legal Roman numeral using *all* of these digits:  
C, C, C, D, I, X, X.
16. [6 pts] Find all values of the digit  $d$  in the 6-digit number  $528,3d4$  that will make the number divisible by 12. Justify your response by demonstrating appropriate divisibility tests.

17. (a) *[3 pts]* List all the factors of 100.

(b) *[4 pts]* True or false: There is a whole number that is both a factor of 100 and also a multiple of 7. Justify your response with an appropriate argument.

18. *[5 pts]* Demonstrate the Prime Number Test in determining whether 739 is prime. State your conclusion.

19. *[8 pts]* Find four factors of the number  $81 \cdot 35^2 \cdot 11$  that are between 200 and 400.

20. (a) *[6 pts]* Use any technique to find the GCD of 375 and 435. Show clear work.

(b) *[6 pts]* Now find the LCM of 375 and 435 using any technique. Show clear work.