Math 210 - Dr. Miller - Exam #1, Version A, Fall 2016 - Thursday, Sept. 29, 2016

This exam is worth 100 points. Follow instructions carefully to receive full credit.

1. [4 pts] Briefly describe the difference between inductive and deductive reasoning.

We are using <u>inductive</u> reasoning when we come to conclusions or make predictions based on (a few) <u>examples</u> or observed data.

We are using <u>deductive</u> reasoning when we apply unchangeable rules of <u>logic</u> to known facts or proven results to draw conclusions.

Left margin annotations:
(-2) no idea of reas. AND basis
(-1) lapse in basis.
(-1) vague
(-2) reversed
(-1) misread

2. [6 pts - 3 each] List the first five terms of a sequence for each description below.

(a) Its third term is 18 and its difference sequence is $+10, -1, +20, -1, +30, -1, \dots$.

$$\underline{9}, \underline{19}, \underline{18}, \underline{38}, \underline{37}$$

$+10 \quad -1 \quad +20 \quad -1$

Left margin annotations:
(-1) diff. list / 18 not 3rd
(-2) 4 terms overall
(-2) Fib-type
(-1) $a_1 = \frac{1}{2}a_2$
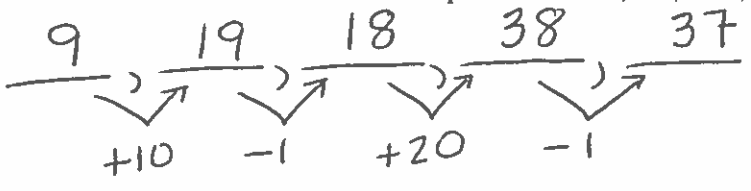
(b) The sequence is arithmetic. Its first term is one third of a_2, and their sum is 80.

$$\underline{20}, \underline{60}, \underline{100}, \underline{140}, \underline{180}$$

$\frac{1}{3}$ of $\quad$ sum = 80
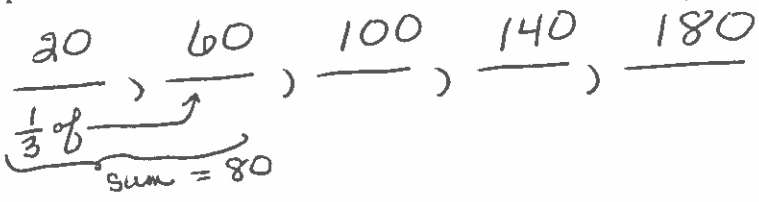
(Guess + check can find the first 2 terms.) ④

3. [8 pts] Find the <u>2105th term</u> of this sequence; show clear work, and briefly <u>explain</u> the reasoning involved. ④

$$8000, \quad 2000, \quad 500, \quad 125, \quad \dots$$

$\div 4 \quad \div 4 \quad \div 4$
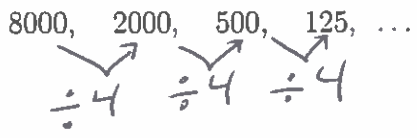
The sequence is geometric, dividing by 4 (or multiplying by $\frac{1}{4}$) to make each new term.

To get to the 2105th term, we need 2104 new terms after the first. So we'll start with the first term 8000 and divide by 4 2104 times.

Answer: $\boxed{8000 \div 4^{2104}}$

or $8000 \times \left(\frac{1}{4}\right)^{2104}$

Left margin annotations:
(-1) bad CR
(0) observed CR
(1) used 2104
(1) $8000 \div 4(2104)$
(2) 2105
(-2) × 2105
(-3) arith
(-2) no discuss / 2104
(-1) × 4 2104
(-1) × 4 2104

4. *[12 pts]* Find the total $561 + 570 + 579 + 588 + \cdots + 4215$. Show work, but you need not explain.

$$561 + 570 + 579 + \cdots + 4215$$
$$+ 4215 + 4206 + 4197 + \cdots + 561$$
$$\overline{\rule{0pt}{1em}\quad 4776 + 4776 + 4776 + \cdots + 4776} \;②$$

$$= 4776 \times \text{how many terms there are} \;②$$
$$= 4776 \times 407 = 1,943,832 \;②$$

Divide to avoid double counting: $\dfrac{1,943,832}{2} = \boxed{971,916}$

⑥
⑥

\# of terms:
$$4215 - 561 = \;② \; 3654 \text{ total distance}$$
$+9$ per ∧new term means $3654 \div 9 = \;② \; 406 \text{ new terms}$

~~add~~ Count first term: $②$ 407 terms altogether.

5. *[3 pts]* Write precisely how to read "$\{32\} \notin \mathbf{N}$" out loud.

⊖ "IN"

The set containing 32 is not an element of the set of natural numbers.

⊖ no mention of 1 person per seat.
⊖ no mention of left-over empty seats
⊖ comparing empty seats to people

6. *[4 pts]* Tommy's mom took him to the opening night of the new Harry Potter movie, and surprisingly, it wasn't sold out. Refer to the characteristics of one-to-one correspondence to explain how Tommy can tell that there are more seats than people at the theater.

⊖ included ref to cour

Every person has one seat, + there are still empty seats. This shows that there are more seats than people.

-3 refer only to count.

7. *[8 pts - 4 each]* For each problem below, list two problem solving strategies that could reasonably be tried in solving it. Justify each claim, telling why you would choose that strategy, but do not actually solve.

(a) The salad bar has 8 different sliced veggies on it today. How many different combinations of one, some, or all of them could you put on a salad?

Make a table or list — to stay organized

Look for/create a pattern — there can be some repetition to how we list the items

Make a diagram — to visualize the options

(marginal notes:) ① G+C weak justif. ⊖ algebra justification "tells how," not "why" ⊝ unclear justif. "how" instead of "why" else where.

(b) In writing the days of the week over and over - Sunday, Monday, Tuesday, ..., Friday, Saturday, Sunday, Monday, Tuesday, ..., what day will appear 775th?

Look for a pattern - there is obvious repetition here

Use direct reasoning — rules and properties of numbers can help

Make a chart/diagram — a calendar arrangement could help to visualize.

(marginal note:) ⊖ G+C

8. (a) *[4 pts]* If possible make up sets C and D containing three elements apiece so that $n(C \cup D) = 7$. If not possible, explain why.

Not possible — with only 3 elements each, the maximum we could get by "dumping the sets together" would be 6, not 7, elements.

(marginal notes:) ⊖ guaranteed ≥6, not 6 max. ⊝ discussed "all different" ① idea of n justif.

(b) *[4 pts]* If possible, make up sets E and F containing two elements apiece so that so that $E \setminus F = \{5\}$. If not possible, explain why.

$$E = \{5, \triangle\}$$
$$F = \{\triangle, \circ\}$$

$$E \setminus F = \{5\},$$
so desired.

(marginal notes:) ② empty sets ② correct sets but saying not possible ② bad singleton answer ② F too big ② E too big ③ disjoint ④ plural ⊖ failed attempt ②

9. For this ENTIRE PROBLEM, let the universal set be $U = \{31, 32, 33, \ldots, 38\}$, and let $A = \{32, 34, 36, 38\}$, $B = \{x \mid x \le 32\}$, $C = \{31, 35, 38\}$, and $D = \{32, 33, 34, 37, 38\}$.

(a) *[3 pts]* Rewrite A using correct set-builder notation.

$$A = \{x \mid x \text{ is even and in the range } 32 \text{ to } 38\}$$

(b) *[2 pts]* How many subsets does C have altogether?

$$2^3 = 8 \text{ subsets}$$

(marginal note:) ⊖ 7

(c) *[4 pts - 1 each]* Identify each statement as true or false. You need not explain, but write clearly to receive full credit.

$B = \{31, 32\}$

  i. $30 \in B$ false
  ii. $\{32\} \in D$ false
  iii. $\{32\} \not\subseteq D$ false
  iv. $B \subseteq D$ false

(d) *[3 pts]* Use correct notation to write all subsets of D that contain 32 but no other even numbers.

−1 lost one
−2 lost 2 or 3
−2 too many

$\{32\}$ $\{32, 37\}$

$\{32, 33\}$ $\{32, 33, 37\}$

(e) *[6 pts]* Use correct notation to write $(D \cap \overline{A}) \cup C$.

−3 reversed

$\overline{A} = \{31, 33, 35, 37\}$

$D \cap \overline{A} = \{33, 37\}$

$(D \cap \overline{A}) \cup C = \{31, 33, 35, 37, 38\}$

(f) *[3 pts]* Use correct notation to write $C \setminus D$.

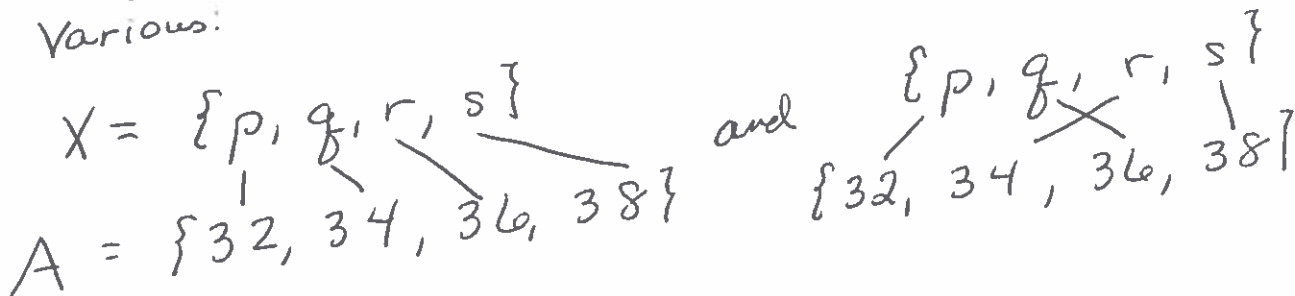$C = \{31, 35, 38\}$

$D = \{32, 33, 34, 37, 38\}$

$C \setminus D = \{31, 35\}$

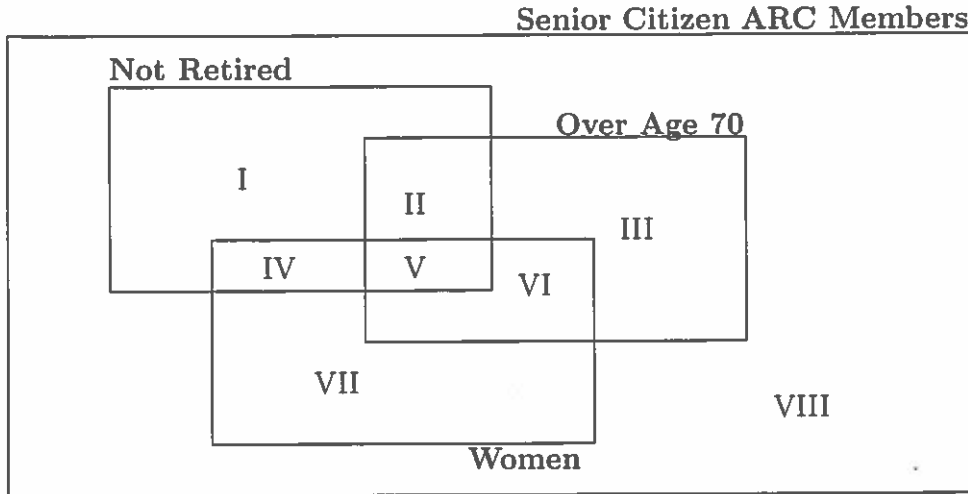(g) *[4 pts]* Make up an appropriate set X and demonstrate two different one-to-one correspondences between X and A.

−1 changed A
−1 one

Various:

$X = \{p, q, r, s\}$

$A = \{32, 34, 36, 38\}$

and $\{p, q, r, s\}$ $\{32, 34, 36, 38\}$

10. Consider the following Venn diagram:

Senior Citizen ARC Members

Not Retired

Over Age 70

I

II

III

IV

V

VI

VII

VIII

Women

(a) *[3 pts]* Describe as fully as possible the people who are in Region II.

(−1) *each false error*

They are men over age 70 who aren't retired and are senior citizen ARC members.

(−1) *each error*

(b) *[2 pts]* List all possible regions where you could put 68-year-old women who aren't retired.

IV

(c) *[2 pts]* Into which region(s) could you put retired women? List all possible.

VI or VII

11. *[3 pts]* An Attribute Game has the following seven labels ONLY: red, green, large, small, triangle, square, hexagon. There is a large green triangle in the left-hand crescent and a large red triangle in the right-hand crescent. What are the possible labels for the two circles, and which is which?

large green △   large red △
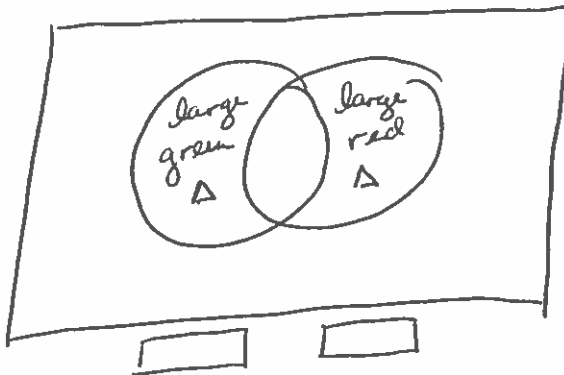
Left
green

Right
red

(Neither can be "large," because we were allowed to put a large shape outside each. Same rules out "triangle.")

12. (a) *[10 pts]* The word arithmetic puzzle below represents a 3-digit number added to another 3-digit number and producing a 4-digit answer. Each letter represents one of the digits 0 through 5 only (no 6s, 7s, 8s, or 9s). Find the value of each letter. Show clear work, and briefly explain your reasoning.

```
    5  2  3
    C  A  R
    5  2  0
 +  C  A  T
 _____
 1  0  4  3
 S  T  I  R
```

(red annotations: ⑤ ⑤ ✓)

(red margin notes: −5 letter = 2 #s ; −3 out of range ; −3 reasoning gives no insight)

T must be 0, since adding it to R leaves R unchanged.

S must be 1, since the only digit that can "carry" when you add 2 digits C+C is 1.

C must be 5 since the only digits allowed that can cause a "carry" are two 5s.

A and I must be 2 and 4 since the only digits in the list that add up, one to create the other, are these.

That leaves R = 3.

0	1	2	3	4	5
T	S	A	R	I	C

(b) *[2 pts]* Clearly state Polya's Four Steps, in order.

1) Understand the problem
2) Devise a plan.
3) Carry it out.
4) Look back.
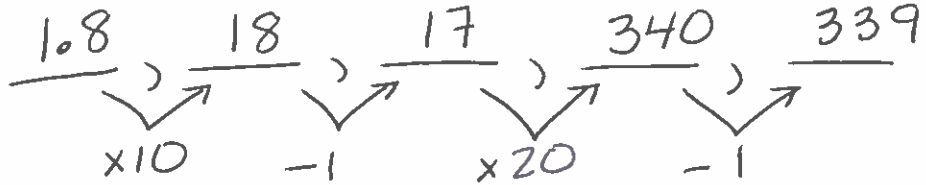
(red margin note: −3 each error)

Math 210 - Dr. Miller - Exam #1, Version B, Fall 2016 - Thursday, Sept. 29, 2016

This exam is worth 100 points. Follow instructions carefully to receive full credit.
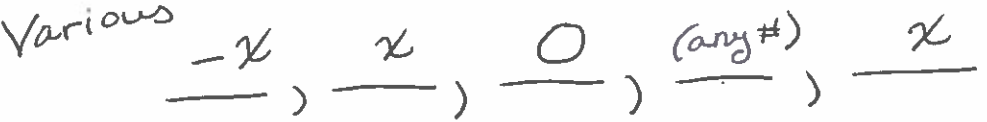
1. [4 pts] Briefly describe the difference between equal and equivalent sets.

Equal sets have exactly the same elements, while equivalent sets simply have the same number of elements.

(left margin annotations):
-3 reworded vague
-1 vague
-1 "same #s"
-1 "same objects, in each"
(need objects, elements, terms)
-1 = two order

2. [6 pts - 3 each] List the first five terms of a sequence for each description below.

(a) Its second term is 18 and its difference sequence is $\times 10, -1, \times 20, -1, \times 30, -1, \dots$.

$$\underline{1.8}, \underline{18}, \underline{17}, \underline{340}, \underline{339}$$

×10   −1   ×20   −1

(left margin annotations):
-1 4 terms
overage
-2 backwards to right
-1 arith error
-2 18 not 2nd
-2 wrong type

(b) The sequence is Fibonacci-type with $a_3 = 0$ and $a_4 \neq 0$.

Various

$$\underline{-x}, \underline{x}, \underline{0}, \underline{(any\,\#)}, \underline{x}$$

3. [8 pts] ④ Find the 2105th term of this sequence; show clear work, and briefly explain the reasoning involved. ④

$$250, \quad 50, \quad 10, \quad 2, \quad \dots$$

÷5 ÷5 ÷5
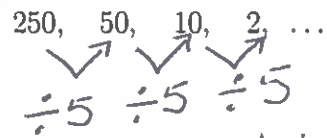
The sequence is geometric with a CR of ÷5 (or ×$\frac{1}{5}$), to make each new term.

To get to the 2105th term, we need 2104 new terms after the first one.

So we'll divide the first term (250) by 5 2104 times.

Answer: $250 \div 5^{2104}$

(or $250 \times \left(\frac{1}{5}\right)^{2104}$).

(left margin annotations):
=1 bad CR
1 observed CR
1 used 2104
1
-2 250÷5(2104)
-3 2105
-4 ×2105
-3 can't no discuss
-1 no discuss 2104
-1 ×5^2104
-1 ×5

4. *[12 pts]* Find the total $561 + 570 + 579 + 588 + \cdots + 4233$. Show work, but you need not explain.

(-1) 2 bad = signs

$$561 + 570 + 579 + \ldots + 4233$$
$$+ 4233 + 4224 + 4215 + \ldots + 561$$
$$\overline{4794 + 4794 + 4794 + \ldots + 4794} \;\text{②}$$

(-1) steps in wrong location

$$= 4794 \times \text{how many terms there are} \;\text{②}$$
$$= 4794 \times 409 = 1,960,746 \;\text{②}$$

Divide to adjust for double-counting:
$$\frac{1960746}{2} = \boxed{980,373} \;\text{①}$$

⑥ # of terms:
$$4233 - 561 = 3672 \quad \text{total distance} \;\text{②}$$

⑥ @ +9 per new term, $3672 \div 9 = 408$ new terms ②

~~Include~~ count first term also: 409 terms in list ② ②

5. *[3 pts]* Write how to read "$B = \{x \mid x \le 32\}$" out loud.

(-0) omitted "all"
(-1) "set"
(-1) out of order
(+1) no / meaning

B is the set containing all x for which x is less than or equal to 32.

6. *[4 pts]* Tommy's mom took him to the opening night of the new Harry Potter movie, and surprisingly, it wasn't sold out. Refer to the characteristics of one-to-one correspondence to explain how Tommy can tell that there are more seats than people at the theater.

(-1) no mention of 1 person per seat
(-1) omit left-over seats

Each person has a seat, but there are still empty seats/seats left over. That means there are more seats than people.

7. *[8 pts - 4 each]* For each problem below, list two problem solving strategies that could reasonably be tried in solving it. Justify each claim, telling why you would choose that strategy, but do not actually solve.

(a) How many different ways are there to express 101 as a sum of three even numbers and one odd number?

Make a list - to stay organized

Look for a pattern - there may be repetition

Use direct reasoning - properties & operations of numbers are involved.

(b) Amara found a bag of pennies at home. She gave one fifth of them to the local charity, then gave 60 pennies to her Girl Scouts. When she was done, she still had 120 pennies in the bag. How many were there to start with?

Work backwards — we know the end + have a chain of events

Write an equation — there's a relationship among amounts in the problem.

8. (a) *[4 pts]* If possible make up sets C and D containing three elements apiece so that $n(C \cap D) = 5$. If not possible, explain why.

Not possible — if each set only has 3 elements to begin with, then they cannot have 5 elements in common.

(b) *[4 pts]* If possible, make up sets E and F containing two elements apiece so that so that $E \setminus F = \{5\}$. If not possible, explain why.

Various:

$E = \{5, 3\}$
$F = \{3, \triangle\}$

$E \setminus F = \{5\}$, as desired.

9. For this ENTIRE PROBLEM, let the universal set be $U = \{31, 32, 33, \ldots, 38\}$, and let $A = \{32, 34, 36, 38\}$, $B = \{x \mid x \leq 32\}$, $C = \{31, 35, 38\}$, and $D = \{32, 33, 34, 37, 38\}$.

(a) *[3 pts]* Rewrite A using correct set-builder notation.

$A = \{x \mid x$ is even and in the range 31 to 38$\}$

(b) *[2 pts]* Are any of the sets $A, B, C,$ or D disjoint? Explain.

$B = \{31, 32\}$

No. Each has something / at least one element, in common with each of the others.

(c) *[4 pts - 1 each]* Identify each statement as true or false. You need not explain, but write clearly to receive full credit.

   i. $30 \in B$ *false*

   ii. $\{32\} \in D$ *false*

   iii. $\{32\} \not\subseteq D$ *false*

   iv. $B \subseteq D$ *false*

(d) *[3 pts]* Use correct notation to write all subsets of D that contain 32 but no other even numbers.

(-1) lost one
(-2) lost 2 or 3
(-2) too many

$\{32\}$

$\{32, 33\}$

$\{32, 37\}$

$\{32, 33, 37\}$

(e) *[6 pts]* Use correct notation to write $(D \cup \overline{A}) \cap C$.

(-2) $(D \cup A) \cap C$
$= \{38\}$

$$\overline{A} = \{31, 33, 35, 37\}$$

$$D \cup \overline{A} = \{31, 32, 33, 34, 35, 37, 38\}$$

$$(D \cup \overline{A}) \cap C = \{31, 35, 38\}$$

(f) *[3 pts]* Use correct notation to write $C \setminus D$.

(-2) idea of deletion

$$C = \{31, 35, \cancel{38}\}$$

$$\setminus D = \{\cancel{32}, \cancel{33}, \cancel{34}, \cancel{37}, \cancel{38}\}$$
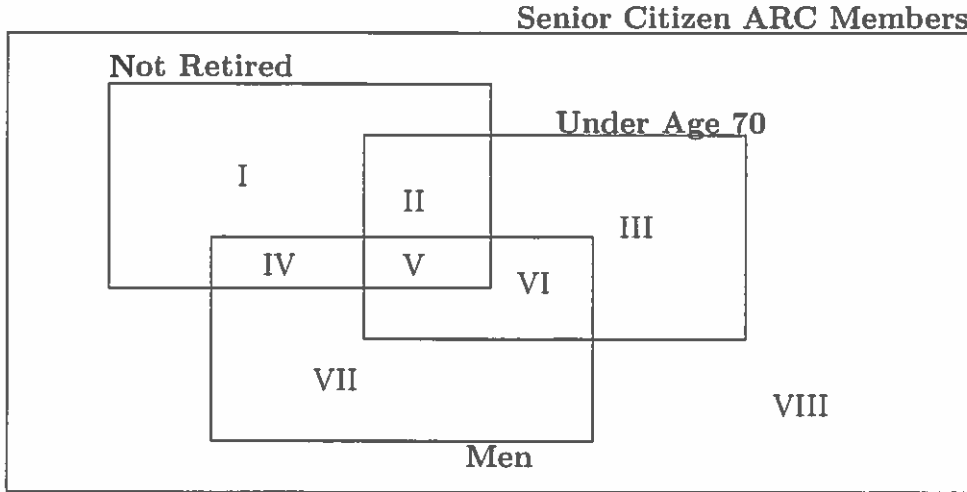
$$C \setminus D = \{31, 35\}$$

(g) *[4 pts]* Make up an appropriate set X and demonstrate two different one-to-one correspondences between X and A.

(-2) half of ONE
(-1) changed A
(-1) one

Various:

$$A = \{32, 34, 36, 38\}$$

$$X = \{p, q, r, s\}$$

or

$$\{32, 34, 36, 38\}$$

$$\{p, q, r, s\}$$

10. Consider the following Venn diagram:

Senior Citizen ARC Members

Not Retired

Under Age 70

I

II

III

IV

V

VI

VII

VIII

Men
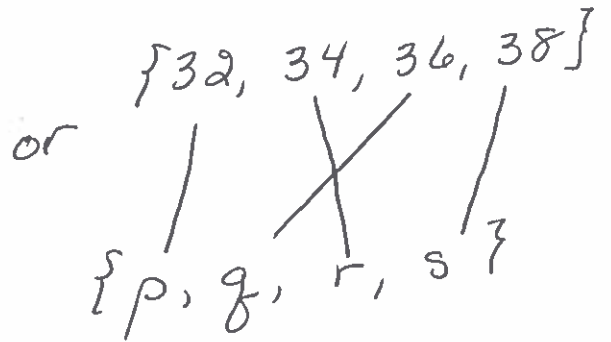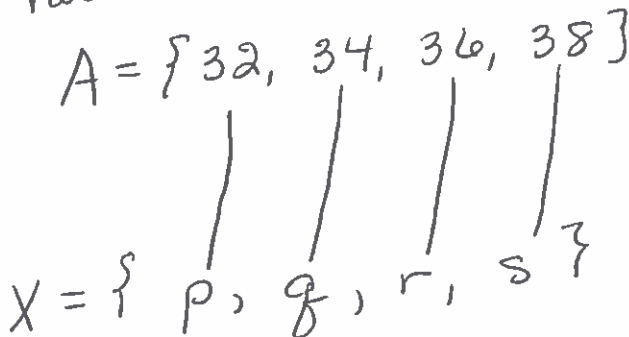
(-1) each error/loss

(a) *[3 pts]* Describe as fully as possible the people who are in Region II.

They are senior citizen ARC members who are women, under age 70, and not retired.

(-1) each error

(b) *[2 pts]* List all possible regions where you could put 75-year-old men who aren't retired.

IV

(c) *[2 pts]* Into which region(s) could you put retired women? List all possible.

III or VIII

11. *[3 pts]* An Attribute Game has the following seven labels ONLY: red, green, large, small, triangle, square, hexagon. There is a large green triangle in the left-hand crescent and a large red triangle in the center lens. What are the possible labels for the two circles, and which is which?

(-2) Right is correct

large green △   large red △

Left
large
or
triangle

Right
red

12. (a) *[10 pts]* How many times is the symbol 2 written if you write down all of the ④ counting numbers from 500 through 1000? Show clear work, and briefly explain your reasoning.

(shorter explan. may be full-credit)

(-2) 95 w/ complete expl.

(-3) lost 520s

Very bad solns deduct from expl.

We could write all those numbers + just count, but that would take far too long. However, we can see a pattern —

502 is the first # that uses a 2, + then there won't be another one until 512, then 522 — ah, it has two 2s! — 532, etc. So #s that end in 2 are:

502, 512, 522, 532, 542
552, 562, 572, 582, 592 } That's ten #s, + it happens again

in the 600s, 700s, 800s, and 900s.

So altogether, 5 groups of 10 #s end in 2.

But some # have a 2 in the middle:

520, 521, 522, 523, 524,
525, 526, 527, 528, 529 } These ten #s give us ten more 2s that aren't counted above,

+ this happens again in the 620s, 720s, 820s, and 920s

So 50 2s are written at the ends of #s,
50 more 2s are written in the middle, for

$$\boxed{100 \ 2s \ \text{altogether}}$$

(b) *[2 pts]* Who came up with the Four Steps studied in this course (spell correctly), and what are they used for?

Polya
They're used for problem solving.