Key  $\frac{24}{24}$

1. [4 pts] Complete this sentence to illustrate the constructivist definition:

$7 \times 3 = 21$ because ...   $7 + 7 + 7 = 21$

2. [8 pts - 2 each] Fill in each blank with the correctly spelled term:

   (a) In the number sentence $13 - 5 = 8$, the answer, 8, is called the

   _difference_.

   (b) In the number sentence $24 \div 8 = 3$, the number 24 is called the

   _dividend_.

   (c) In the number sentence $9 \times 4 = 36$, the numbers 9 and 4 are called the

   _factors_

   while the answer, 36, is called the

   _product_.

3. [6 pts] Circle the computation that is impossible, then use your choice of definition or division model to clearly explain why.

② some use of a model or defn.

(-5) mult.

(-2) $3 \div 0 \neq 0$.

$\boxed{3 \div 0}$

Defn - If $3 \div 0$ had an answer, we'd need answer $\times 0 = 3$. That's impossible.

Rep. Subt. Model - You can't separate 3 objects into piles of 0 each.

$0 \div 3$

Sharing model - you can't separate 3 objects into 0 groups.

4. [6 pts - 3 each] For each word problem below, write the complete number sentence it requires and the name of the model (i.e., "take away") that it demonstrates.

   (a) Rani has 3 colors of icing and 5 types of sprinkles to use in decorating her cupcakes. How many different combinations can she make?
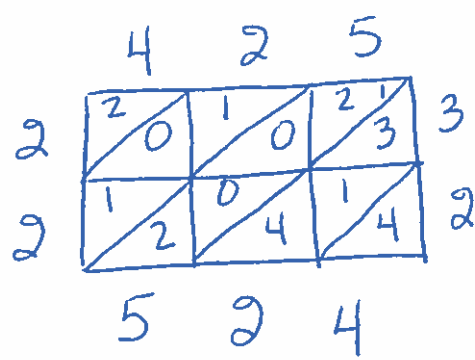
   $3 \times 5 = 15$   Cartesian product

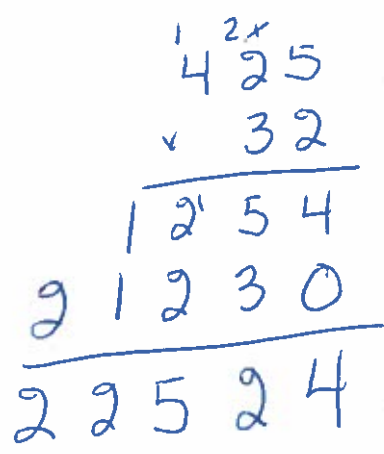   (b) Tran has 18 tomato plants to put in rows of 6 each. How many rows can he plant?

   $18 \div 6 = 3$   repeated subtraction.

$17^3 \quad 17^2s$
$17s$
$| \ // \ ones$

5. *[6 pts]* Convert, showing clear work:   $2038_{seventeen}$ = _____ten

$$2 \cdot 17^3 + 0 \cdot 17^2 + 3 \cdot 17 + 8 \cdot 1$$

$$= 9,885_{ten}$$

6. (a) *[3 pts]* List the three numerals that immediately follow $5TT_{twelve}$ in base twelve.

$5TE$ , $5EO$ , and $5E|$

(b) *[3 pts]* List the three numerals that immediately precede $5TT_{twelve}$ in base twelve.

$5T9$ , $5T8$ , and $5T7$

7. *[6 pts]* Refer to the concept of place value in explaining why we do not use the digit "4" in base four.

⊖-4  PV but saying only 0,1,2,3.

⊖-2  Discuss 10 next.

In base four, there is already a position (or "place") reserved for recording fours, so we don't need a digit also.

8. *[10 pts]* Add entirely in base twelve, showing clear work:   $2T4 + 881 + 909 + 1362 + 4T44$

$$
\begin{array}{r}
{}^2\ {}^1 \\
2\ T\ 4 \\
8\ 8\ 1 \\
9\ 0\ 9 \\
1\ 3\ 6\ 2 \\
+\ 4\ T\ 4\ 4 \\
\hline
7\ T\ 5\ 8_{twelve}
\end{array}
$$

9. *[10 pts]* Multiply entirely in base six using any algorithm you choose, showing clear work:  $425_{six} \times 32_{six}$
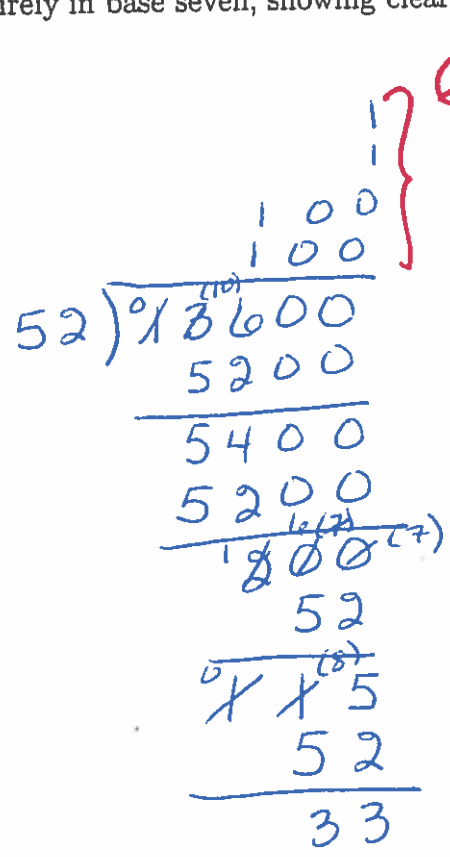
(-8) base ten lattice

Lattice method:
```
      4   2   5
   ┌───┬───┬───┐
   │2/ │1/ │2/ │ 3
 2 │/0 │/0 │/3 │
   ├───┼───┼───┤
   │1/ │0/ │1/ │ 2
 2 │/2 │/4 │/4 │
   └───┴───┴───┘
     5   2   4
```

$$\boxed{22524_{six}}$$

Standard algorithm:
```
    1 2×
   4 2⁵5
 ×    3 2
 ───────────
   1 2 5 4
 2 1 2 3 0
 ───────────
 2 2 5 2 4
```

10. *[12 pts]* Divide entirely in base seven, showing clear work:  $13600_{seven} \div 52_{seven}$

(-6) — in base 10.

(-3) each × in base 10

(-3) 15 at end, not 115 (lost front!)

(-8) bad positions.

(-2) offset.

```
        1
        1 0 0
        1 0 0
          (10)
 52 ) 1̷ 3̷ 6 0 0
      5 2 0 0
      ─────────
      5 4 0 0
      5 2 0 0
      ─────(7)
        1̷ 8̷ 0̷ 0̷ (7)
          5 2
        0̷ (8)
        1̷ 1̷ 5
          5 2
        ─────
          3 3
```

$$\boxed{202 \text{ rem } 33}$$

11. *[8 pts]* Subtract entirely in base sixteen using the "balancing" algorithm, showing clear work:    $6986_{sixteen} - 72A_{sixteen}$

(hand-written, red margin notes) -6 regroup. 720 / -3 720 / -3 6992 / -3 6992

$$6986 \xrightarrow{\ +6\ } \qquad 698C$$
$$-\ 72A \xrightarrow{\ +6\ } \qquad -\ 730$$
$$\overline{\qquad\qquad} \qquad\qquad \overline{625C}\ _{sixteen}$$

12. *[6 pts]* Is the set $\{2, 3, 5, 6\}$ closed under multiplication? Explain.

(red margin notes) -3 left unfinished set. 3·2=6 / -3 yes, 3·2=6

No.    3 and 5 are in the set, but 3×5 is not.

13. *[12 pts - 3 each]* Finish the number sentence to demonstrate each required property. Do not demonstrate any other properties than the one requested.

$$8(7+6) + (5+4) = \underline{\ \ (?)\ \ }$$

(red margin note) -2 CP of add.

(a) The Commutative Property of Multiplication

$$(7+6)\cdot 8\ +\ (5+4)$$

(red margin notes) -2 ·1 / -3 +1

(b) The Identity Property of Addition

$$0+\ 8(7+6)\ +\ (5+4)$$

(red margin note) -2 left out at regroup.

(c) The Associative Property of Addition

$$\Big(8(7+6) + 5\Big) + 4$$

(d) The Distributive Property of Multiplication over Addition

$$8\cdot 7 + 8\cdot 6\ +(5+4)$$