

1. [9 pts - 3 each] Circle the larger Roman numeral in each pair:

(a)

$CDXLIII = 443$

$CDXCIV = 494$

(b)

$LVIII = 58$

$XCVIII = 98$

(c)

$CMI = 901$

$DCCI = 701$

2. [6 pts] Create the smallest possible Roman numeral from these digits: ~~C~~, ~~C~~, ~~D~~, ~~I~~, ~~M~~, ~~X~~, ~~X~~.

-2 each if illegal  
-1 if illegal

MCDXCIX

3. (a) [8 pts] Find all values of the digit  $d$  in the number  $152d2$  that make this number divisible by 4. Briefly explain your reasoning in a sentence.

$d = 1, 3, 5, 7, \text{ or } 9.$

The last 2-digit part of the large number must be a multiple of 4. That can only be 12, 32, 52, 72, or 92.

(b) [5 pts] Demonstrate the test for divisibility by 11 on the number  $31526$ . State your conclusion.

subtract 
$$\begin{array}{r} 3+5+6 = 14 \\ 1+2 = 3 \\ \hline \end{array}$$

11 is a multiple of 11, so

yes

31526 is too.

4. [8 pts] Use the definition of divides to decide whether this statement is true or false: "30 divides 15." Show clear work and thorough explanation.

(-2) forgot LW  
 (-4) correct for  $15 \mid 30$   
 (-2) division  $15 \div 30$

$$30 \cdot \square = 15$$

The box cannot be filled with a whole number, so the statement is false.

5. [6 pts] Show work in using the Divisibility-of-a-Sum Theorem to determine whether 20,999,999,979 is divisible by 21.

$$\begin{array}{r} 21,000,000,000 \text{ is divisible by } 21 \\ 21 \text{ is too} \\ \hline 20,999,999,979; \text{ their difference must be also.} \end{array}$$

6. [5 pts] In a 1-600 Sieve of Eratosthenes, what is the largest number that will produce any "crossing out"?

(-3) only  $\sqrt{}$

$$\sqrt{600} \approx$$

7. [8 pts] True or false: If a number is divisible by 12 and 15, then it must be divisible by  $12 + 15$ . Justify your answer.

(-4) no/bad ex  
 (5) "3"

False. Sixty is divisible by both 12 and 15, but not by 27.

44  
44

8. (a) [8 pts] How many different factors does the number  $3^5 \cdot 5^2 \cdot 11^4 \cdot 17^9$  have in all?

5, 2, 4, 9  
6 · 3 · 5 · 10  
900

(b) [12 pts] Find four of them that are between 30 and 200.

$17 \cdot 3 = 51$   
 $17 \cdot 9 = 153$   
 $17 \cdot 5 = 85$

$5^2 \cdot 3 = 75$   
 $11^2 = 121$   
etc.

- 33
- 55
- 45
- 187
- 85
- 135
- 99

-5  $10^3$   
 $14^2$

9. [8 pts] Use any technique to find the LCM (only!) of the numbers  $10^3 \cdot 7^2$  and  $14^2 \cdot 3$ .

$LCM = 2^3 \cdot 3 \cdot 5^3 \cdot 7^2$   
 $= 147,000$

$2^3 \cdot 5^3 \cdot 7^2$       $2^2 \cdot 3 \cdot 7^2$

-5

10. [8 pts] Use listing to find the GCD of 40 and 108.

- 1, 40
- 2, 20
- 4, 10
- 5, 8

- 1, 108
- 2, 54
- 3, 36
- 4, 27
- 6, 18
- 9, 12

GCD = 4

11. [8 pts] The LCM of  $a$  and 960 is 4800 and their GCD is 32. Find  $a$ . Show clear work.

$$\frac{4800 \cdot 32}{960} = \frac{a \cdot 960}{960}$$