Math 235 - Dr. Miller - HW #1: Logic Review - **Due Monday, 1-28-2025, by 4pm in my physical mailbox**

HW handouts in this course don't leave room for your work, so always work on your own paper, **leave room for MY feedback comments**, and staple the question sheet to the front when finished.

1. What can be said - if anything - about the truth value of the statement form $(P \leftrightarrow Q) \rightarrow (P \vee R)$ in each case below? Describe your reasoning in a sentence or two. (Do NOT make full truth tables.)
   (a) $P$ and $R$ are both false.
   (b) $P$ and $Q$ have different truth values.
   (c) $P \leftrightarrow Q$ is true but $P \rightarrow R$ is false.

2. Again consider the statement form $(P \leftrightarrow Q) \rightarrow (P \vee R)$. If $P \wedge Q$ is true, is there a fixed truth value for $R$ that will make the above statement form false, or do we not have enough information? Explain your reasoning.

3. Negate the statements below; use simplest logical form (SLF) when meaningful. (See below.)
   (a) $\sin A = 1$ and $\cos A \leq 00$.
   (b) Either of $2x + y = 0$ or $y \geq 3$ implies that $x \leq 1.5$.
   (c) If $x$ or $y$ is even, then $xy$ is even.
   (d) $\frac{b^2}{a} \in \mathbf{Z}$ if and only if $\frac{b}{a} \in \mathbf{Z}$.
   (e) $n > n^2$ if $0 < n < 1$.
   (f) $\sin A$ and $\cos A$ have the same sign only if $A$ is in Quadrant I or in Quadrant III.
   (g) There exists a function $f$ for which, if $x > 0$, then $f(x) < 0$ or $f(x) > 1$.
   (h) There are real numbers $x$ and $y$ where $y$ is negative and $x^2 + y^2 = 1$.
   (i) For each $x \in \mathbf{R}$, there is $y \in R$ for which $xy > 1$ and $y$ is irrational.
   (j) For every $\epsilon > 0$, there is a number $\delta > 0$ satisfying $\epsilon + \delta < 0$.

   (*) SLF means we:
   - Avoid double negatives like "is not non-zero." (That becomes "IS zero.")
   - Avoid generic "it's not the case that..." lead-in phrases.
   - Fully negate "and/or" statements using de Morgan's Laws.
   - Fully negate quantifiers: never keep half-negations like "there does not exist..." or "not all...."

4. For each conditional statement below, do two things:
   (I) Identify its hypothesis, written as a stand-alone sentence (that is, with no conditional words remaining: for example, "Silver is a cat," not "<u>if</u> Silver is a cat.")
   (II) Write the indicated variation (converse, inverse, contrapositve) using the form required.

   (a) If $a$ and $b$ have different signs, then $ab < 0$. — For (II), write the converse in if-then form.
   (b) $ab$ being positive implies that $|a + b| = |a| + |b|$. — For (II), write the inverse in if-then form.
   (c) $a$ can only be a multiple of $b^2$ if $a$ is a multiple of $b$. — For (II), write the contrapositive in if-then form.
   (d) It is necessary that $c$ be negative for $a^4b^2c$ to be negative. — For (II), write the converse using "only if."
   (e) $|a + b| = |a| + |b|$ only if $a$ and $b$ have the same signs. — For (II), write the inverse using "sufficient."
   (f) $a^3$ is positive if $a$ is positive. — For (II), write the contrapositive using a "trailing if."