Math 235 - Dr. Miller - HW#3, Spring 2025 - **Due by 4pm Friday, 2-7-2025**

1. Prove rigorously, using a direct proof (possibly cases/WLOG):

   (a) Prop. - Let $x, y, z \in \mathbf{Z}$. If at least one of them is divisible by 13, then xyz is too.

   (b) Prop. - Let $x, y \in \mathbf{Z}$. If they have different odd remainders on division by 5, then $5 \mid xy + 2$.

   (c) Prop. - Let $x, y, z \in \mathbf{Z}$. If exactly two of them have the same parity, then $xy + yz + xz$ is also of that same parity.

2. Write the logical equivalence that governs "or conclusion" style proof, then use a Discrete Math-type truth table to confirm the equivalence. State in a sentence what behavior in the table actually shows that these statement forms are equivalent.

3. Prove rigorously, using "or conclusion" style:

   (a) Prop. - Let $x, y \in \mathbf{R}$ with $y \neq 0$. If $\dfrac{5+x}{y}$ is rational, then x is rational or y is irrational.

   (b) Prop. - Let $n \in \mathbf{Z}$. If $4 \nmid n$, then $n \div 4$ has remainder 1 or else $4|(3n^2 + n - 2)$.

   (c) Prop. - Let $a, b \in \mathbf{Z}$. If $3|ab$, then $3|a$ or $3|b$. (Caution: the algebra here needs special care and focus.)

4. Consider this Proposition: *Let $x, y \in \mathbf{R}$. If $x + y \neq 0$, then $x \neq 0$ or $y \neq 0$.*

   (a) Proving it by contrapositive is a better choice than direct proof; why? (Don't say: "It's easier.")

   (b) Write a rigorous proof by contrapositive for the Proposition.

5. Prove rigorously by contrapositive:

   (a) Prop. - Let $x, y \in \mathbf{Z}$. If xyz is even, then at least one of x, y, or z is even.

   (b) If $x + y \notin \mathbf{Q}$, then at least one of x or y is irrational.