Math 235 - Dr. Miller - HW#4, Spring 2025 - **Due by 4pm Friday, 2-14-2025**

1. In DP #2, I pointed out that in our course, a proof "launch" is the entire start of a proof up to one meaningful sentence beyond the NTS. Write launches for the following propositions, in the style indicated. Do **NOT** worry about whether the proposition is actually true or not, and do **NOT** continue the proof beyond giving a meaningful first sentence after the NTS!

   (a) A proof by contrapositive for: *Let* $x, y \in \mathbf{Z}$. *If* $3|x^2$ *or* $3|xy$, *then* $3|x$ *or* $3|y$.

   (b) An "or conclusion" style proof for: *Let* $x, y \in \mathbf{Z}$. *If* $3|x^2$ *or* $3|xy$, *then* $3|x$ *or* $3|y$.

   (c) A direct proof of: *Let* $x \in \mathbf{Z}$. *If* x *has odd remainder on division by* 6, *then* x *is odd.*

   (d) (Careful!) A proof by contradiction for: *Let* $x, y \in \mathbf{Z}$. *Then* xy *is even if* x *is even or* y *is even.*

   (e) (Careful!) A proof by contrapositive for: *Let* $x, y \in \mathbf{Z}$. *Then* xy *is even only if* x *is even or* y *is even.*

2. We know that proof by contrapositive is governed by the fact that a conditional statement is logically equivalent to its contrapositive. Create a Discrete Math-type truth table to confirm the equivalence. State in a sentence what behavior in the table actually shows that these statement forms are equivalent.

3. (a) Rigorously prove using "or-conclusion" style:

   Prop. - Let a, b, and m be integers. If $12m + 1 \leq 2a + 3b$, then $b \geq 4m - 1$ or $a \geq 2$.

   (b) Now use a different proof style to prove the same result.

4. Rigorously prove by any meaningful method; also label each proof with the style you chose.

   (a) DELETED

   (b) Prop. - Let a be a real number. If $2a + 1 \in \mathbf{Q}$ or $\frac{a+1}{2} \in \mathbf{Q}$, then a is rational.

   (c) Prop. - A circle has center $(2, 4)$. If $(0, 3)$ is outside the circle, then $(3, 1)$ is outside the circle.

   (d) Prop. - Let $x \in \mathbf{Z}$. If $8 \not| \, x^2 - 1$, then x is even.

5. Rigorously prove by contradiction:

   (a) Prop. - There is no smallest odd integer.

   (b) Prop. - All points on $y = 6 - x$ lie outside the circle centered at $(-3, 1)$ with radius 2.

   (c) Prop. - $\sqrt{12}$ is irrational.

   (d) Prop. - $\sqrt[5]{3}$ is irrational.