Math 235 - Dr. Miller - Exam #1 - 9/26/03

1. Make up examples of sets A, B, and C satisfying the following conditions.
   (a) $A \in B$, $B \in C$, and $A \subseteq C$.
   (b) $A \in \mathcal{P}(\mathbf{Q})$, $B \subseteq \mathcal{P}(\mathbf{Q})$, and $|A| = |B| = 3$

2. Consider these sets: $X = \{1, \{2\}, 3, \{4\}\}$, $Y = \{1, 2, \{1, 2\}\}$, $W = \{\{1\}, \{2\}, 1, 2\}$, and $V = \{\emptyset, \{1\}\}$.
   (a) Using correct notation, determine $\mathcal{P}(V)$.
   (b) Using correct notation, determine $X \setminus W$.
   (c) Using correct notation, determine $\mathcal{P}(Y) \cap W$.

3. For each collection of sets A_i and index set I, find $\cup_{i \in I} A_i$ and $\cap_{i \in I} A_i$. Show work, but you need not prove.
   (a) $A_i = \{i^2\}$, $I = \mathbf{Z}$
   (b) $A_i = [-\frac{1}{n}, \frac{1}{n}] \cup [1 - \frac{1}{n}, 1 + \frac{1}{n}]$, $I = \mathbf{N}$

4. Make up a collection of *distinct* sets A_i for which $\cup_{i \in \mathbf{N}} A_i = [0, 2]$ and $\cap_{i \in \mathbf{N}} A_i = \{0\}$.

5. Determine via a completed truth table whether $(P \vee \sim Q) \wedge P$ is logically equivalent to $\sim (P \Longrightarrow Q)$. Clearly state your conclusion.

6. Rewrite each statement below entirely in symbolic form:
   (a) Every negative real number is less than its own square.
   (b) There are natural numbers x and y for which $x - y$ and $x + y$ have different signs.
   (c) If x is even, then x^2 is a multiple of 4.

7. Verbally restate #6c using the phrase "only if."

8. Symbolically negate each of the following, expressing your response in simplest form.
   (a) $\forall x \in (0, \infty), \exists y \in \mathbf{R} \ni y^2 < x$

(b) $\forall x \in \mathbf{R}, x < 0$ or $\sqrt{x} \geq 0$

(c) $\exists x, y \in Z \ni x > y \Longrightarrow x^2 > y^2$

9. Prove using mathematical induction: $(1 + \frac{1}{1}) \cdot (1 + \frac{1}{2}) \cdots (1 + \frac{1}{n}) = n + 1$ for all natural numbers $n \geq 2$.