Math 235 - Dr. Miller - Exam #2a - 10/29/03

This portion of the exam is worth 47 points.

1. *[4 pts]* Identify the full hypothesis and conclusion of each statement below:
   (a) A group is abelian only if its Cayley table is symmetric.
   (b) Every square is a parallelogram.

2. *[3 pts]* Let x and y be natural numbers. If you wished to prove the following statement by contrapositive, what would you assume, and what would you need to show?
   If x and y are both odd, then $xy > 2$.

3. *[4 pts]* Illustrate the <u>definition</u> of the term "divides" in confirming or refuting each of the following statements.
   (a) 6 is a multiple of -24.
   (b) 7 is a divisor of 0.

4. *[10 pts]* Prove that $x^2 - 3$ is even if and only if x is odd.

5. *[10 pts]* Prove that if $x + y$ is odd, then x and y are of different parity.

6. *[8 pts]* Prove that if $a|b$ and $a|c$, then $a|(b + c)$.

7. *[8 pts]* Prove that if $2|x$, then $x^2 \equiv 0 \bmod 4$.

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1. *[3 pts]* Make up a statement that is vacuously true, then tell how you know.
2. *[2 pts]* Name two numbers that are congruent modulo 15, telling how you know.
3. *[4 pts]* Consider the following proof:
   Let x and y both be rational. Then $x = \frac{m}{n}$ and $y = \frac{p}{q}$ for some integers m, n, p, q where $nq \neq 0$. Then $x - y = \frac{mq-np}{nq}$, which is rational. What result has been proved?
4. *[4 pts]* lemma reassemblage
5. *[10 pts]* set proof #1
6. *[10 pts]* set proof #2
7. *[10 pts]* divides proof
8. *[10 pts]* congruence proof