

1. [12 pts] Prove: The quotient $\frac{p}{q}$ of any nonzero rational number p and any irrational number q is irrational.
2. [12 pts] Explain thoroughly why there exists a number that is irrational and strictly between 0 and $1/2$.
3. [8 pts] Disprove: The quotient of two irrational numbers is irrational.
4. [8 pts] Disprove: If $x, y \in \mathbf{R} \setminus \{0\}$ with $x > y$, then $1/x < 1/y$.
5. [20 pts - 4 each] Let $f : A \longrightarrow B$ be a relation. Circle the best response to complete each statement below.
 - (a) " $f(x) = f(y) \implies x = y$ " means that f is ...

a function	not a function	one-to-one	not one-to-one	onto	not onto
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 - (b) " $\forall b \in B, \exists a \in A$ such that $(a, b) \in f$ " means that f is ...

a function	not a function	one-to-one	not one-to-one	onto	not onto
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 - (c) "The graph of f fails the Vertical Line Test" means that f is ...

a function	not a function	one-to-one	not one-to-one	onto	not onto
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 - (d) " $1/2 \in A$ and $f(1/2) \neq f(2/4)$ " means that f is ...

a function	not a function	one-to-one	not one-to-one	onto	not onto
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 - (e) " $3 \in B$ and $\nexists a \in A$ with $f(a) = 3$ " means that f is ...

a function	not a function	one-to-one	not one-to-one	onto	not onto
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6. [20 pts - 10 each] Make up a function $f : \mathbf{R} \longrightarrow \mathbf{R}$ that is ...
 - (a) not onto - justify your claim
 - (b) not one-to-one - justify your claim
7. [20 pts] Prove carefully that the function $f(x) = 1/x$ is a bijection from $[1, \infty)$ to $(0, 1]$.