Math 235 - Dr. Miller - Final Exam - 12/8/03

All cell phones and pagers must be turned **OFF**. This exam is worth 150 points.

1. *[6 pts]* Make up sets A, B, and C for which $A \in B$, $C \in \mathcal{P}(A \times B)$, and $|A| = |B| = |C| = 2$.
2. *[10 pts]* Determine $\cap_{i \in I} A_i$ and $\cup_{i \in I} A_i$ when $A_i = \{\frac{1}{i}\} \cup (2 - \frac{1}{i}, 3 + \frac{1}{i}]$ with index set $I = \mathbf{N}$.
3. *[6 pts]* Negate the following statement, writing your answer in simplest symbolic form.

$$\forall x \in \mathbf{R}, \exists y, z \in \mathbf{N} \text{ such that } y > x \Longrightarrow (yz \leq xz \text{ or } x > z)$$

4. *[6 pts]* Fill in each blank with a correct natural number, then explain how the *definition* of the specified concept justifies your response:
   (a) $24 \mid$ ____
   (b) $116 \equiv$ ____ mod 5
5. *[8 pts - 2 each]* Let $f : A \longrightarrow B$ be a relation. Circle the best response to complete each statement.
   (a) "$x = y \Longrightarrow f(x) = f(y)$" means that f is ...

   a function   not a function   one-to-one   not one-to-one   onto   not onto

   (b) "$3 \in B$ and $\exists a_1 \neq a_2 \in A$ with $f(a_1) = f(a_2) = 3$" means that f is ...

   a function   not a function   one-to-one   not one-to-one   onto   not onto

   (c) "$2 \in A$ and $\nexists b \in B$ with $f(2) = b$" means that f is...

   a function   not a function   one-to-one   not one-to-one   onto   not onto

   (d) "Every element of B occurs as a second coordinate in F" means that f is ...

   a function   not a function   one-to-one   not one-to-one   onto   not onto

6. *[12 pts - 4 each]* Disprove each statement:
   (a) $\forall x, y \in \mathbf{R}$, $x < y \Longrightarrow x^2 < y^2$.
   (b) Every infinite set is uncountable.
   (c) The quotient of any two rational numbers is rational.

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7. (a) *[10 pts]* Prove by any method: $x + y$ is even *only if* x and y have the same parity.
   (b) *[2 pts]* What type of proof did you use above?
8. *[10 pts]* Prove: If $A \subseteq B$, then $A \cup B = B$.
9. *[15 pts]* Prove: $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$.
10. *[15 pts]* Prove: $x^2 \equiv 1 \bmod 3$ if and only if $x \equiv 1$ or $2 \bmod 3$.
11. *[15 pts]* Prove: The sum of any rational number p and irrational number q is irrational.
12. *[15 pts]* Prove: $\frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{n(n+1)} = \frac{n-1}{2(n+1)}$ for all natural numbers $n \geq 2$.
13. *[20 pts]* Create a bijection from $[-5, 4]$ to $[16, 28]$; prove that your function truly is a bijection.