

HW handouts in this course don't leave room for your work, so always work on your own paper, **leave room for MY feedback comments**, and staple the question sheet to the front when finished.

1. Negate the statements below; use simplest logical form (SLF) when meaningful. (See below.)

- (a) $\sin A = 1$ and $\cos A \leq 0$.
- (b) Either of $2x + y = 0$ or $y \geq 3$ implies that $x \leq 1.5$.
- (c) If x or y is even, then xy is even.
- (d) $\frac{b^2}{a} \in \mathbf{Z}$ if and only if $\frac{b}{a} \in \mathbf{Z}$.
- (e) $n > n^2$ if $0 < n < 1$.
- (f) $\sin A$ and $\cos A$ have the same sign only if A is in Quadrant I or in Quadrant III.
- (g) There exists a function f for which, if $x > 0$, then $f(x) < 0$ or $f(x) > 1$.
- (h) There are real numbers x and y where y is negative and $x^2 + y^2 = 1$.
- (i) For each $x \in \mathbf{R}$, there is $y \in \mathbf{R}$ for which $xy > 1$ and y is irrational.
- (j) For every $\epsilon > 0$, there is a number $\delta > 0$ satisfying $\epsilon + \delta < 0$.

(*) SLF means we:

- Avoid double negatives like "is not non-zero." (That becomes "IS zero.")
- Avoid generic "it's not the case that..." lead-in phrases.
- Fully negate "and/or" statements using de Morgan's Laws.
- Fully negate quantifiers: never keep half-negations like "there does not exist..." or "not all..."

2. For each conditional statement below, do two things:

- (I) Identify its hypothesis, written as a stand-alone sentence (that is, with no conditional words remaining: for example, "Silver is a cat," not "if Silver is a cat.")
- (II) Write the indicated variation (converse, inverse, contrapositive) using the form required.

- (a) If a and b have different signs, then $ab < 0$. — For (II), write the converse in if-then form.
- (b) ab being positive implies that $|a + b| = |a| + |b|$. — For (II), write the inverse in if-then form.
- (c) a can only be a multiple of b^2 if a is a multiple of b . — For (II), write the contrapositive in if-then form.
- (d) It is necessary that c be negative for a^4b^2c to be negative. — For (II), write the converse using "only if."
- (e) $|a + b| = |a| + |b|$ only if a and b have the same signs. — For (II), write the inverse using "sufficient."
- (f) a^3 is positive if a is positive. — For (II), write the contrapositive using a "trailing if."

3. Determine whether each attempt below satisfies all characteristics of a good definition. If any are lacking, state which ones, and repair the definition to fix those issues. (You may need to review, but you can accept *polygon*, *side*, *side length*, and *interior angle* as known concepts.)

- (a) A triangle out of all polygons has exactly 3 sides.
- (b) A scalene triangle is a polygon that has all its side lengths different.
- (c) An equilateral is if it has all its side lengths equal.
- (d) A right triangle is a triangle containing exactly one interior right angle.
- (e) An obtuse triangle is when it contains exactly one interior obtuse angle.
- (f) An acute triangle is a triangle containing exactly one interior acute angle.

continued on back

4. Write clear, complete definitions of the following terms, subject to our criteria. (You may need to look these up first.)

- (a) complex number
- (b) square root
- (c) parabola

5. Rewrite each description below to fit our criteria for being a good definition.

- (a) *Perpendicular lines intersect at 90 degree angles.*
- (b) *For a function f to be periodic, there has to be a real number p such that $f(x + p) = f(x)$ for all $x \in \mathbf{R}$.*
- (c) *Even numbers happen when you can divide by 2 and not get a remainder.*

6. In the list of shapes below, B, C, and E are flurbs while A, D, and F are not. Use this identification to write a definition for a flurb that meets our criteria.

