

1. Determine whether each attempt below satisfies all characteristics of a good definition. If any are lacking, state which ones, and repair the definition to fix those issues. (You may need to review, but you can accept *polygon*, *side*, *side length*, and *interior angle* as known concepts.)

- (a) A triangle out of all polygons has exactly 3 sides.
- (b) A scalene triangle is a polygon that has all its side lengths different.
- (c) An equilateral is if it has all its side lengths equal.
- (d) A right triangle is a triangle containing exactly one interior right angle.
- (e) An obtuse triangle is when it contains exactly one interior obtuse angle.
- (f) An acute triangle is a triangle containing exactly one interior acute angle.

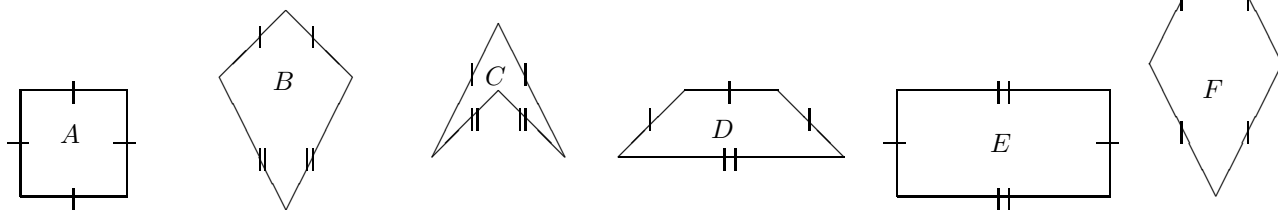
2. Write clear, complete definitions of the following terms, subject to our criteria. (It's okay to look these up and then adjust results that you find.)

- (a) complex number
- (b) square root
- (c) parabola

3. Rewrite each description below to fit our criteria for being a good definition.

- (a) *Perpendicular lines intersect at 90 degree angles.*
- (b) *For a function  $f$  to be periodic, there has to be a real number  $p$  such that  $f(x + p) = f(x)$  for all  $x \in \mathbf{R}$ .*
- (c) *Even numbers happen when you can divide by 2 and not get a remainder.*

4. In the list of shapes below, B, C, and E are flurbs while A, D, and F are not. Use this identification to write a definition for a flurb that meets our criteria.



continued on the back

The next two problems address the idea of interpretations for an axiomatic system. Both use the attached map of the US, which I got through Facebook years ago. The map shows 2014 data for the three most common surnames (last names) in each of the 50 states and the 2-letter postal abbreviation for that state. I have posted a color image AND an Excel spreadsheet (to help you sort!) in D2L reading Content.

5. Using the map, create this interpretation for our undefined terms in Incidence Geometry:

(\*) Points are surnames: Miller, Johnson, Lee, etc.      (\*) Lines are states: PA, OH, NV, etc.

(\*) “Lie on” means that name occurs in the surname list for that state, so Miller I PA.

- (a) Find three lines that Anderson is incident with.
- (b) Find a point that is NOT collinear with Miller. Tell in a short sentence how you know.
- (c) List (in alphabetical order by abbreviation) any lines that are parallel to PA.
- (d) Find a line that intersects NV in exactly two points. Tell in a short sentence how you know.
- (e) Are Johnson, Anderson, and Smith collinear? Tell in a short sentence how you know.
- (f) We’re used to thinking that two lines are equal if they “contain” exactly the same points, but that attempt at a definition actually requires that our undefined terms be SETS and ELEMENTS. That’s not the case here. For us, different states ARE different lines, regardless of the points involved. List (alphabetically by abbreviation) all DISTINCT lines that are incident with Smith, Johnson, and Miller simultaneously.
- (g) Find a line that is parallel to the smallest number of other lines. Tell how you know, including listing those other lines alphabetically by abbreviation.
- (h) Find a point that is collinear with the largest number of other points. Tell how you know. (You may list - alphabetically - if you like.)
- (i) Find a point that is collinear with the smallest number of other points. Tell how you know.

6. Now let’s change our interpretation, still using the map:

(\*) Points are STATES this time.      (\*) Lines are SURNAMES.

(\*) But “lie on” still means that name occurs in the surname list for that state, it’s just that the notation changes order: so PA I Miller.

This kind of thing where we trade the roles of points and lines from an interpretation is called creating the DUAL of the original interpretation.

- (a) How many lines does OH lie on?
- (b) Find a point that is NOT collinear with OH. Tell in a short sentence how you know.
- (c) List (in alphabetical order) three lines that are parallel to Miller. Tell how you know.
- (d) Can you find a set of three collinear points? Tell in a short sentence how you know.
- (e) Can you find a pair of lines that intersect in exactly two points? Tell how you know.
- (f) List (alphabetically) all DISTINCT lines that are incident with PA and TX simultaneously.
- (g) Find a line that is parallel to the smallest number of other lines. Tell how you know.
- (h) Find a point that is collinear with the smallest number of other points. Tell how you know.