Math 304 - Dr. Miller - HW #4, Spring 2025 - **Due by 4pm Friday, 2-14-2025, in my office or VSC 200 mailbox**

1. We've analyzed given IG interpretations to see whether they meet or fail each Incidence Axiom, but this question reverses that task. For each part below,

   (*) Points are members of the set $A = \{ S,\ I,\ L,\ V,\ E,\ R \}$.

   (*) Lines will be subsets of A that *you* create.

   (*) "Lie on" will mean set membership: P I ℓ if $P \in \ell$.

   In each part, **YOU** must create a collection of lines that make the indicated axioms hold/fail. Also, every point must be used in each part, though how often it's used is up to you.

   (a) IA #1 holds but IA #2 and #3 fail.

   (b) IA #2 holds but IA #1 and #3 fail.

   (c) IA #3 holds but IA #1 and #2 fail.

2. Suppose we have an interpretation in which S, R, and U are the only points, and there are no lines at all. Decide whether each Incidence Axiom passes or fails, and justify clearly. (Be careful with the logic - it's quite subtle here.)

3. Rigorously prove these Incidence Theorems, following the rules of using ONLY information that comes earlier in the axiomatic system. (For instance, to prove 2.6.3, you CANNOT use 2.6.4, even though you did encounter 2.6.4 on earlier HW.)

   (a) Theorem 2.6.3

   (b) Theorem 2.6.9

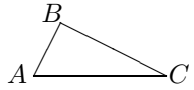
continued on back

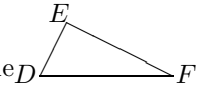
4. Below are several interpretations for the undefined terms in Incidence Geometry. Some are from earlier HW; others are new. For each interpretation, clearly and thoroughly explain whether each Parallel Postulate holds or fails. (Your justifications should do more than just state what the Postulate - or its negation - says. Give extra info about how you can tell it meets or fails its conditions.)

   (a) Referring to the surname map from HW #2:

   (*) Points are surnames: Miller, Johnson, Lee, etc. (*) Lines are states: PA, OH, NV, etc.

   (*) "Lie on" means that name occurs in the surname list for that state, so Miller I PA.

   (b) From HW #3, "points" are the letters A, B, C, D, E, and F. There are two lines: one is the triangle 

and the other is the triangle 

. "P lies on ℓ" means P is a vertex of triangle ℓ.

   (c) (New) Let n be a fixed integer that's greater than 3. The set of points is the collection of distinct symbols A_1, A_2, ..., A_n. The set of lines consists of all sets of the form $\{A_i, A_j, A_k\}$ where $i \neq j \neq k (\neq i)$. "Lie on" means "is an element of."

   (d) From HW #3, points are individual real numbers. Lines are non-empty intervals of the form (a, b) where $a, b \in \mathbf{R}$. (Infinity is NOT a number!!!) A point lies on a line if that point belongs to the interval.

   (e) (New) Points are individual real numbers. Lines are intervals of the form (a, ∞) where $a \in \mathbf{R}$. A point lies on a line if that point belongs to the interval.

   (f) From HW #3, points are members of $S = \{0, 1, 2, 3, 4\}$. Lines are also members of S. P lies on ℓ means $2P + \ell > 3$. (You may want to recopy and use your incidence tables from HW #3.)